Name solutions

April 10, 2013

ECE 311 Exam 3 Spring 2013

Closed Text and Notes

1) Be sure you have 10 pages plus the equation sheets.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(10 pts) 1. Fill in the table with the units, and whether a vector or scalar, for the indicated quantities.

| | units | Vector or scalar? |
|---|---|---|
| Magnetic field intensity | $\frac{A}{m}$ | vector |
| Magnetic flux density | $\frac{Wb}{m^2} = T$ | vetor |
| Magnetic flux | $Wb = Vs$ | scalar |
| Magnetization | $\frac{A}{m}$ | vector |
| Inductance | $H = \frac{Wb}{A}$ | scalar |

(10 pts) 2. Everywhere there is a magnetic flux density of $\mathbf{B}=2.5\,\hat{\mathbf{a}}_x$ T. There is a circular conduction loop in the xy plane. At t = 0 a current starts flowing in the loop such that $\mathbf{m}=5\,\hat{\mathbf{a}}_z$ Am$^2$. The loop is free to rotate.

A) What is the total force on the current loop for t > 0?

$$\vec{F} = \oint I\,d\vec{l} \times \vec{B} = -I \oint \vec{B} \times d\vec{l}$$

$$= -I\vec{B} \times \oint d\vec{l} \quad \text{since } \vec{B} \text{ is constant}$$

$$= 0 \quad \text{since } \oint d\vec{l} = 0$$

B) What is the magnetic dipole moment for t >> 0?

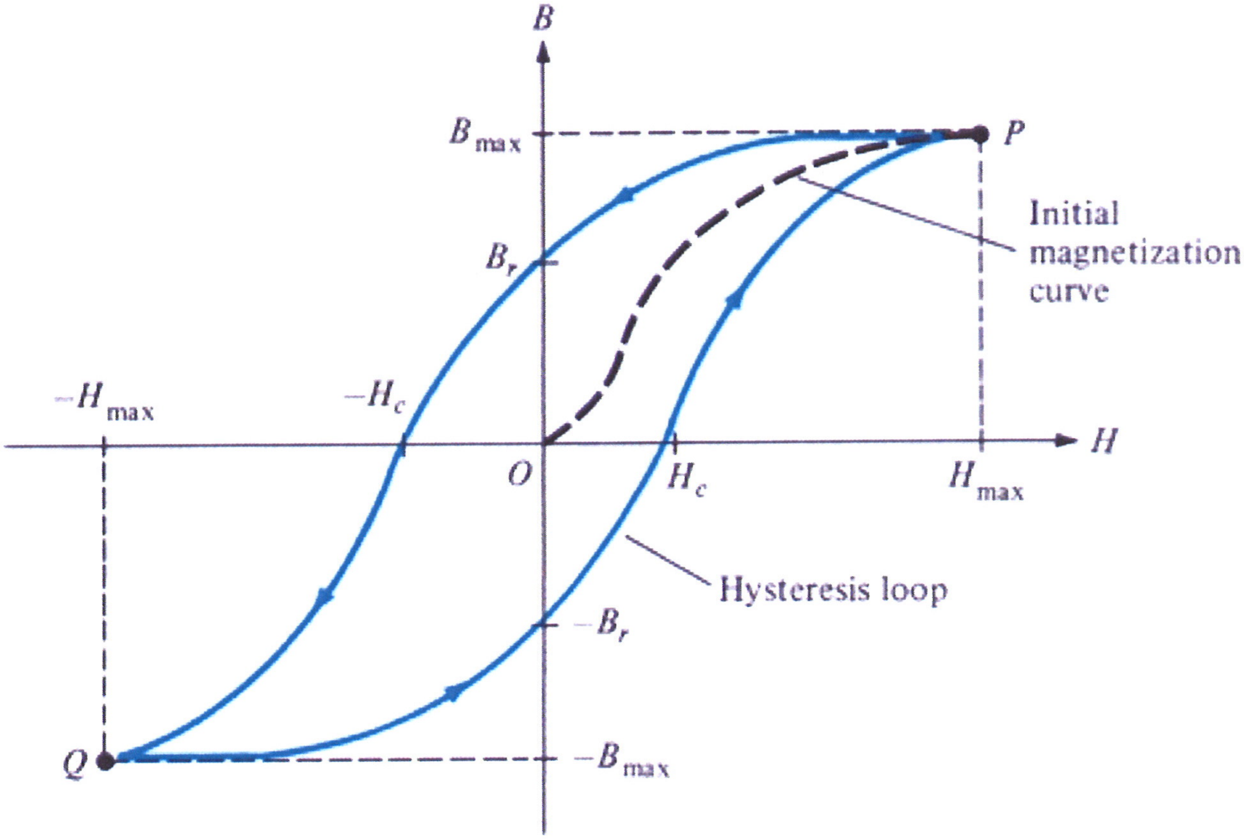
$$\vec{T} = \vec{m} \times \vec{B}$$

the loop will rotate till $\vec{T} = 0$

so,

$$\vec{m} = 5\,\hat{a}_x \text{ Am}^2$$

Problems 3 and 4 refer to the shown hysteresis curve for a ferromagnetic material.




(5 pts) 3. The magnetization field saturates in a ferromagnetic material because

A) all the atomic dipoles are already aligned to the applied magnetic field.

B) thermal energy in the material does not allow any more alignment of the atomic dipoles.

C) of a limit on the size of the applied field.

D) an internal field is generated to oppose the applied field.

(5 pts) 4. The slope of the B-H curve for H > Hmax is

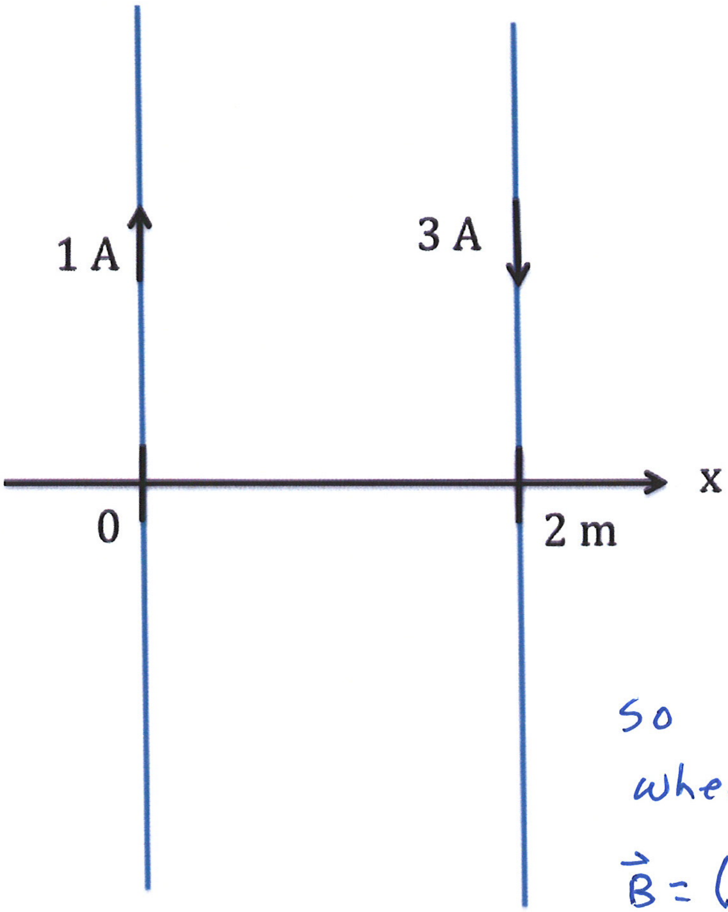
A) infinite

B) zero

C) not constent

D) $\mu_o$

(10 points) 5. Two infinite wires are parallel, in the xz plane, and with currents flowing as shown. What is the force per unit length experienced by the wire at x = 2 m? Use $\mu_0 = 4\pi \times 10^{-7}\,\frac{H}{m}$




The $I = 1A$ wire generates the magnetic field intensity,

$$\oint \vec{H} \cdot \vec{dl} = I_{encl.} = 1A$$

$$H\,2\pi\rho = 1A$$

$$\vec{H} = \frac{1A}{2\pi\rho}\,\hat{a}_\phi$$

so,

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 (1A)}{2\pi\rho}\,\hat{a}_\phi$$

So the $\vec{B}$-field at $x = 2m$, where $\rho = 2m$, is

$$\vec{B} = \left(4\pi \times 10^{-7}\frac{H}{m}\right)\frac{1A}{2\pi\,(2m)}\,\hat{a}_\phi$$

$$\vec{B} = 10^{-7}\,\frac{HA}{m^2}\,\hat{a}_\phi = 10^{-7}\,\frac{Wb}{m^2}\,\hat{a}_\phi$$

Calculate the force on the segment of the $I = 3A$ wire from $z$ to $z + l$

$$\vec{F} = \int I\,\vec{dl} \times \vec{B} = \int_z^{z+l} (3A)(-dz\,\hat{a}_z) \times 10^{-7}\frac{Wb}{m^2}\,\hat{a}$$

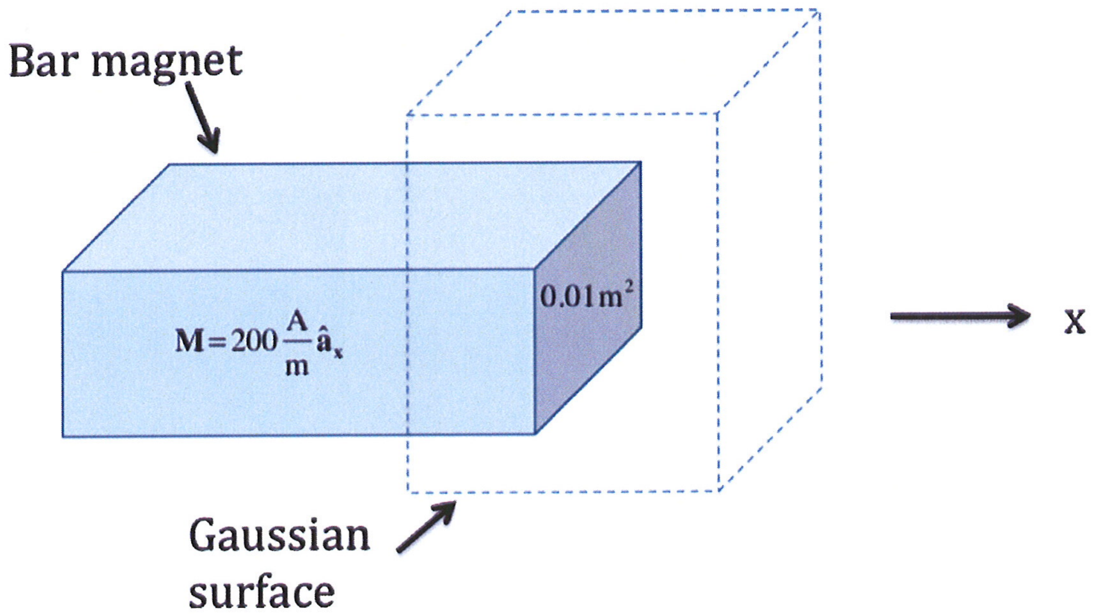
$$\vec{F} = (3A)\left(10^{-7}\frac{Wb}{m^2}\right)\int_z^{z+l} dz\,\hat{a}_x = \left(3\times 10^{-7}\,\frac{A\,Wb}{m^2}\right) l\,\hat{a}_x$$

$$\text{force per unit length} = \frac{\vec{F}}{l} = 3\times 10^{-7}\,\frac{A\,Wb}{m^2}\,\hat{a}_x$$

$$= 3\times 10^{-7}\,\frac{N}{m}\,\hat{a}_x$$

$$\frac{A\,Wb}{m^2} = \frac{A\,V\,s}{m^2} = \frac{\frac{C}{s}\,\frac{J}{C}\,s}{m^2} = \frac{J}{m^2} = \frac{Nm}{m^2} = \frac{N}{m}$$

(12 pts) 6. Shown is a bar magnet with magnetization $\mathbf{M}=200\frac{\text{A}}{\text{m}}\hat{\mathbf{a}}_x$. What is $\oint \mathbf{H} \bullet \mathbf{dS}$ over the Guassian surface shown, which completely encloses the right end of the magnet? The right end of the magnet has area $0.01\text{m}^2$.



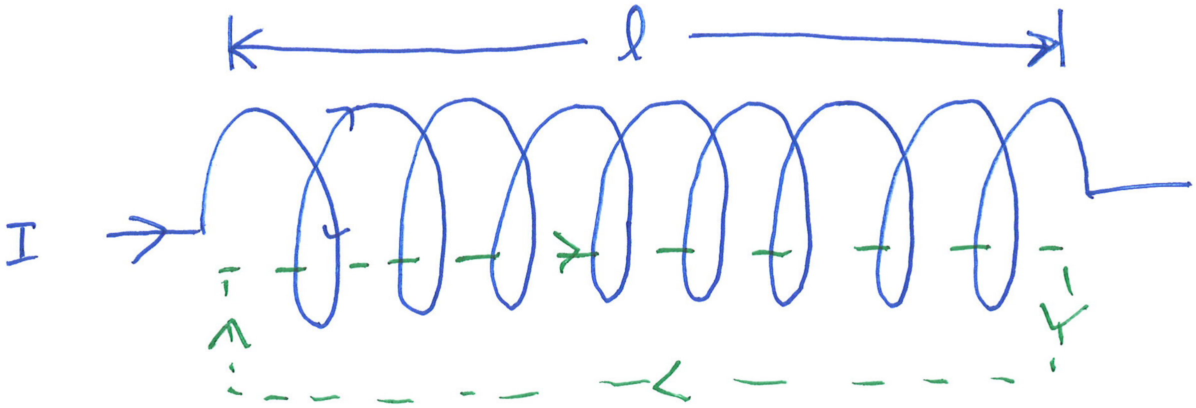

$$\oint \vec{B}\cdot d\vec{s} = 0 \quad \text{and} \quad \vec{B} = \mu_0(\vec{H}+\vec{m})$$

so $$\oint \vec{B}\cdot d\vec{s} = \oint \mu_0(\vec{H}+\vec{m})\cdot d\vec{s} = 0$$

$$\oint \vec{H}\cdot d\vec{s} = -\oint \vec{M}\cdot d\vec{s} = -(200\hat{a}_x)\cdot(-0.01m^2\hat{a}_x) = 2Am$$

$$\oint \vec{H}\cdot d\vec{s} = 2Am$$

(12 pts) 7. A tightly wound solenoid has N turns, is of length l, and radius a. If l >> a, determine the equation for the inductance of this solenoid.




$$\oint \vec{H}\cdot d\vec{\ell} = I_{encl} = NI$$

$$H\ell = NI$$

$$H = \frac{NI}{\ell} \quad \text{inside the solenoid}$$

$$\simeq 0 \quad \text{outside}$$

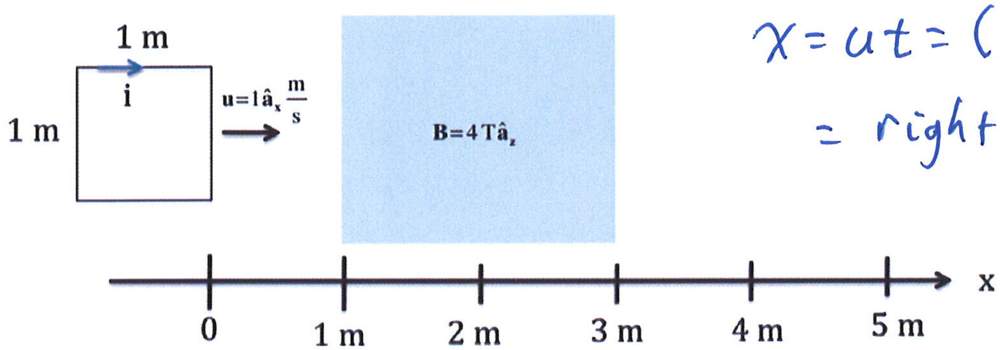
$$B = \mu_0 H = \frac{\mu_0 N I}{\ell} \quad \text{inside the solenoid}$$

$$\Psi = BS = \frac{\mu_0 N I}{\ell}\pi a^2$$

$$\lambda = N\Psi = \frac{\mu_0 N^2 I \pi a^2}{\ell}$$

$$L = \frac{\Psi}{I} = \frac{\mu_0 N^2 \pi a^2}{\ell}$$

(12 pts) 8. A resistive loop has total resistance of 2 $\Omega$, is 1 m x 1 m, in the xy plane, and moving with velocity $\mathbf{u}=1\hat{\mathbf{a}}_x \frac{\text{m}}{\text{s}}$. At t = 0 the left edge of the loop is at x = 0. Shown is a 2 m x 2 m magnetic field region with $\mathbf{B}=4\text{T}\hat{\mathbf{a}}_z$ that the loop will encounter. With the current defined in the clockwise direction as shown, determine the current flowing in the loop as a function of time.




$x = ut = (1\frac{m}{s})t$

$=$ right edge of loop

$t < 1s$ loop has not entered B-field, $i = 0$

$1s < t < 2s$ loop will be entering B-field

Area in B-field $= (x - 1m)(1m) = (ut - 1m)(1m) = (1\frac{m}{s}t - 1m)(1m)$

$\Psi_{out} = (4\frac{Wb}{m^2})(1\frac{m^2}{s}t - 1m^2)$

$\frac{d\Psi_{out}}{dt} = 4V$

$\oint_{cw} E \cdot dl = iR = i(2\Omega) = -\left(\frac{d\Psi_{in}}{dt}\right) = \frac{d\Psi_{out}}{dt} = 4V$

$i = 2A$

$2s < t < 3s$ loop is entirely in B-field $i = 0$

$3s < t < 4s$ loop is leaving B-field

Area in B-field $= [3m - (x - 1m)]\,1m = [4m^2 - xm] = [4m^2 - ut\,m]$

$= [4m^2 - (1\frac{m^2}{s})t]$

$\Psi_{out} = (4\frac{Wb}{m^2})[4m^2 - (1\frac{m^2}{s}t)] = 16\,Wb - 4\frac{Wb}{s}t$

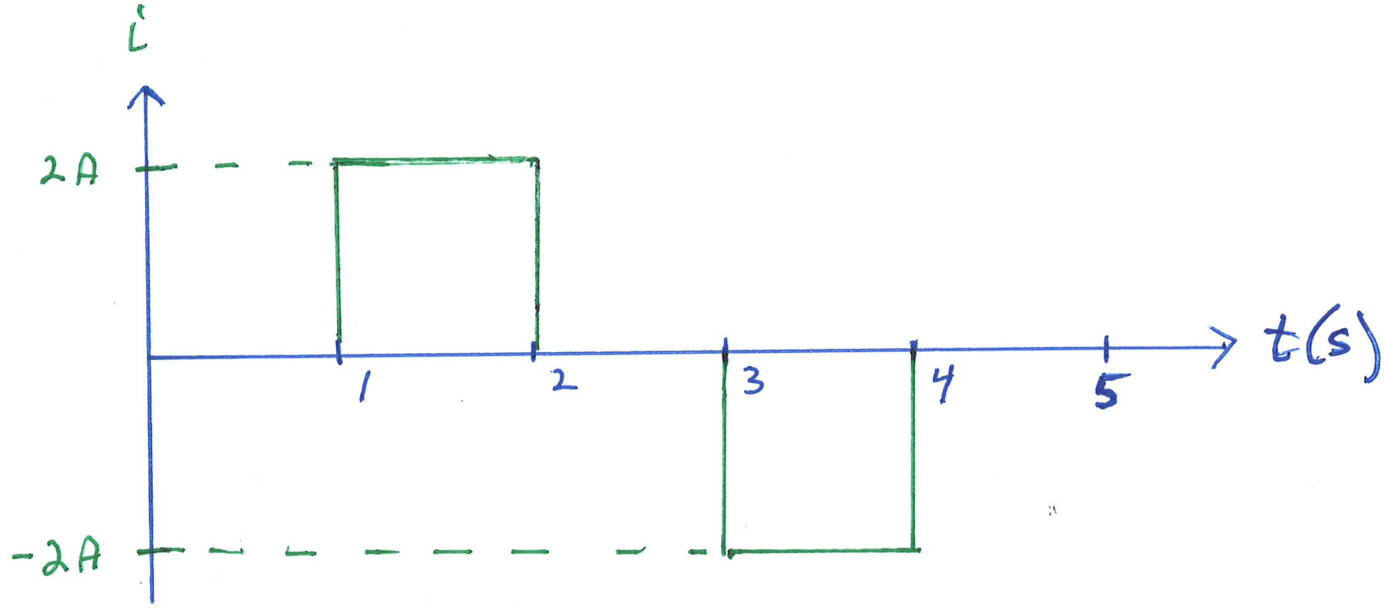
$\frac{d\Psi_{out}}{dt} = -4V$

$$\int_{cw} \vec{E} \cdot \vec{dl} = iR = -\frac{d\Psi_{in}}{dt} = \frac{d\Psi_{out}}{dt} = -4V$$

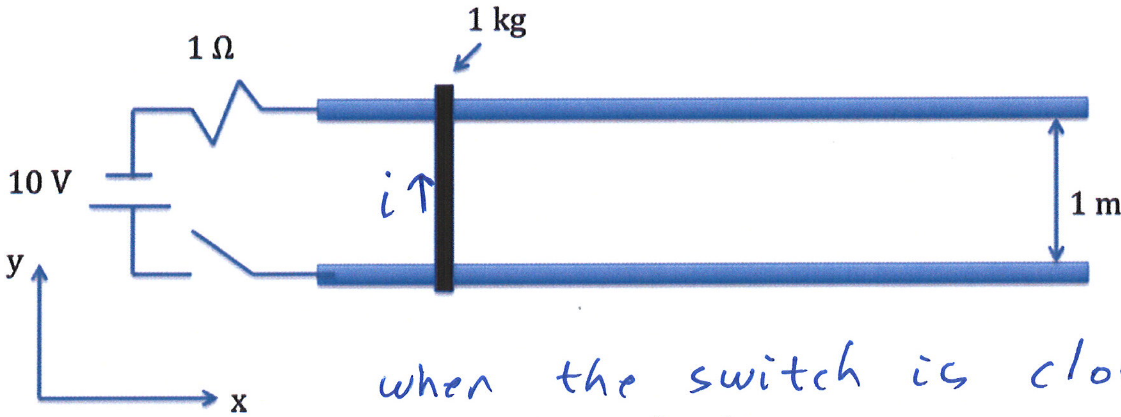
$$i(2\Omega) = -4V$$

$$i = -2A$$

$4s < t$ loop is out of B-field, $i = 0$



(12 pts) 9. A 1 kg sliding bar is on the rail system shown. If the magnetic flux density is everywhere $\mathbf{B}=5\hat{\mathbf{a}}_z$ T, what is the force on the sliding bar? Assume the resistance of the rails and the sliding bar is negligible.




when the switch is closed

$$i = \frac{V}{R} = \frac{10V}{1\Omega} = 10A$$

$$\vec{F} = \int I d\vec{l} \times \vec{B} = \int_0^{1m} (10A)\left[ dy\,\hat{a}_y \times 5\hat{a}_z \frac{Wb}{m^2} \right]$$

$$= 50 \frac{A\,Wb}{m^2} \int_0^{1m} dy\;\hat{a}_x = \left(50 \frac{A\,Wb}{m^2}\right)(1m)\,\hat{a}_x$$

$$= 50\,N\,\hat{a}_x$$

$$\frac{A\,Wb}{m} = \frac{A\,Vs}{m} = \frac{\frac{C}{S}\frac{J}{C}S}{m} = \frac{J}{m} = \frac{Nm}{m} = N$$

(12 pts) 10. One thing we have ignored with the rail system in problem 9 is that as the bar accelerates, there will be a changing flux in the circuit. This will result in an Vemf that will oppose the 10V, reducing the current and reducing the force on the bar. When Vemf = 10 V the force will have gone to aero on the bar. So the bar will have reached a maximum velocity. What is this maximum velocity for the rail system of problem 9?

$$\text{area} = (x)(1\,m)$$

$$\Psi_{\text{out of page}} = \left(5\frac{Wb}{m^2}\right)(x)(1m) = \left(5\frac{Wb}{m}\right)x$$

$$\frac{d\Psi}{dt}_{\text{out of page}} = \left(5\frac{Wb}{m}\right)\frac{dx}{dt} = \left(5\frac{Wb}{m}\right)u$$

$$\int_{cw} \vec{E}\cdot\vec{dl} = V_{emf} = -\left(\frac{d\Psi}{dt}_{\text{into the page}}\right) = 10V$$

$$10V = \frac{d\Psi}{dt}_{\text{out of page}} = \left(5\frac{Wb}{m}\right)u$$

$$u = 2\frac{Vm}{Wb} = 2\frac{Vm}{Vs} = 2\frac{m}{s}$$